

MISSING AND QUENCHED GAMOW TELLER STRENGTH

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Abstract

Gamow-Teller strength functions in full $(pf)^8$ spaces are calculated with sufficient accuracy to ensure that all the states in the resonance region have been populated. Many of the resulting peaks are weak enough to become unobservable. The quenching factor necessary to bring into agreement the low lying observed states with shell model predictions is shown to be due to nuclear correlations. To within experimental uncertainties it is the same that is found in one particle transfer and (e,e') reactions. Perfect consistency between the observed $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ peaks and the calculation is achieved by assuming an observation threshold of 0.75% of the total strength, a value that seems typical in several experiments

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Since the time of the pioneering (p,n) experiments [1], [2], and the more recent (n,p) ones [3], [4] it has been possible to know the full Gamow Teller strength functions of many nuclei. The most striking result is that a large fraction of the theoretically expected sum rules for the $\sigma\tau$ operators, S_+ and S_- , is not visible. The precise amount may be difficult to assess, in particular because calibration discrepancies with beta decay measures [5], [6], but there is no doubt that it is substantial and a reduction by a factor 0.6 of S_+ and S_- is currently accepted as standard. This number is obtained through two different channels. One is the Ikeda sum rule $S_+ - S_- = 3(N-Z)$, which is model independent provided we do not introduce non-nucleonic degrees of freedom -and we will not. Therefore the strength difference cannot be quenched, i.e. suppressed. It is *missing* but it must be somewhere [7].

The other indication comes from the well defined, isolated peaks seen in β decays which are about a factor 0.6 weaker than predicted by the most accurate shell model calculations available [8], [9]. Here we can speak of *quenching* because the data demand it.

In section I we will calculate complete strength functions that suggest that many states must be unobservable. In section II we decompose the model independent sum rule in a way that makes apparent that quenching originates in nuclear correlations. In section III we give the reasons to expect that only about 50% of the S_- sum rule for ^{48}Ca is observed [10].

I. To understand how the strength distributes among daughter states we rely on the method proposed by Whitehead [11] and now quite popular [12], [13], [14]. We work in the full pf shell with the KB3 interaction [15], [16], and obtain an exact eigenstate of the target $|i\rangle$ *in this model space*. Then we define states $|S_{\pm}\rangle = \sigma\tau|i\rangle$ whose norms are the sum rules S_{\pm} , and we use them as pivots (i.e. starting states) in a Lanczos tridiagonal construction. After I iterations we obtain I+1 eigensolutions and the amplitudes of the pivot in each of them determine their share of strength. The situation at I=50 is shown in fig. 1 for $^{48}\text{Ca}(p,n)^{48}\text{Sc}$, i.e. $|i\rangle$ is the $(pf)^8$ T=4 ground state and the pivot is projected to keep only T=3 states. The first 4 spikes correspond to converged eigenstates whose position

and strength will not change as I increases. The others should be viewed as doorways which will split as we evolve. By construction the first 101 (i.e. $2I+1$) moments of the exact distribution are given by the spikes. If we were to compare with experiments with poor resolution but infinite detection power this would be more than sufficient and we could replace each peak by a gaussian to obtain a smooth function. Since infinite sensitivity is not available we have to know in detail how the strength splits and before comparing with data, eliminate the states below the detection threshold. In fig. 2 we show how this could be done by pushing to 700 iterations, which guarantees convergence for all the states below 10.5 MeV. We shall examine later plausible values for the threshold.

In ^{48}Sc the m-scheme matrices are $1.4 \cdot 10^5$ dimensional and there are 8590 $J=1$ $T=3$ states. To guard against numerical errors each new Lanczos vector must be spin and isospin projected and orthogonalized with respect to the preceeding ones. On a IBM-3090 about to retire, ANTOINE [17] can cope with about 100 iterations per hour for this problem.

To analyze a situation in which the density of levels around the resonance is much higher, we select the β decay of ^{48}Mn to ^{48}Cr [18], [19], which reaches the region where the calculated strength with standard quenching is still a factor 2 larger than the observed one [9]. The $J=4$ $T=1$ ground state can go to $J=3,4,5$ and $T=0,1,2$ daughters. Since iterations must be done for each of these separately and since the m-scheme dimension is now $2 \cdot 10^6$, exact calculations with large I can become very heavy. They were done for $I=45$ and it was checked that in the region of interest, configurations $(1f_{7/2})^{8-t}(2p_{3/2}, 1f_{5/2}, 2p_{1/2})^t$ with $t \leq 3$ are sufficient [9]. The corresponding results are shown in figs. 3 and 4 for $I=50$ and 300 which makes clear why we could be spared the effort of an exact calculation with full convergence. In the blow-up in fig. 5 we see that 81.5% of the strength is distributed among the peaks whose share is less than 1% of the total. Pushing further the number of iterations and increasing the size of the spaces could only increase the dilution of much strength into an unobservable background.

II. To understand the origin of the quenching effect we start by writting the target eigenstate $|\bar{1}\rangle$ as a 'dressed' model state $|i\rangle$

$$|\bar{1}\rangle = |i\rangle + \sum_j |j\rangle \langle j|\hat{A}|i\rangle \quad (1)$$

where the j states are outside the model space and \hat{A} is the correlation operator. Now we separate $\sigma\tau$ as:

$$\sigma\tau = (\sigma\tau)_m + (\sigma\tau)_r \quad (2)$$

where $(\sigma\tau)_m$ contains the contribution of the model space (i.e. in our examples pf orbits) and $(\sigma\tau)_r$ contains all others. The total sum rule state can be split accordingly as:

$$\frac{\sigma\tau_\alpha}{\langle \bar{1}|\bar{1}\rangle} |\bar{1}\rangle = |s_\alpha\rangle = |s_{\alpha m}\rangle + |s_{\alpha r}\rangle, \quad \alpha = \pm \quad (3)$$

By using exactly the same arguments that lead to Ikeda's sum rule we have:

$$S_- - S_+ = 3(n_m - z_m) + 3(n_r - z_r) = (S_- - S_+)_m + (S_- - S_+)_r, \quad (4)$$

where n_m , z_m , n_r and z_r are expectation values of number operators, for which obviously $n_m + n_r = N$ and $z_m + z_r = Z$.

Intuitively it is clear $(\sigma\tau)_m|\bar{1}\rangle$ is a state in the model space for the daughter nucleus while $(\sigma\tau)_r|\bar{1}\rangle$ will produce one outside that space. The result is true in leading order of perturbation theory and we propose it as a good approximation. (To be more precise demands information about \hat{A} [20]).

The consequences are very pleasing because now eq.(4) can be interpreted as a clean separation of two contributions: one from the model space and one from outside. The first is then:

$$(S_- - S_+)_m = 3 \langle \bar{1}|\hat{n}_m - \hat{z}_m|\bar{1}\rangle = 3 \langle i|\hat{n}_m - \hat{z}_m|i\rangle \cdot (0.7) \quad (5)$$

(we use \hat{n} and \hat{z} to distinguish operators from expectation values)

The factor 0.7 comes from the (d,p) data of Vold *et al* [21] and is consistent with the occupancies near the Fermi level obtained in (e,e') scattering [22], [23], [24]. Quenching

therefore originates in deep correlations that reduce by about 40% the discontinuity at the Fermi surface.

Although the precise form of the renormalized $\sigma\tau$ operator acting in the model space is in principle complicated, to satisfy eq.(4) it is sufficient to use $\sqrt{0.7}(\sigma\tau)_m$, which is standard practice, except that the factor is in general $\sqrt{0.6}$. In view of experimental and theoretical uncertainties the two factors are most probably compatible. Furthermore, these arguments establish nuclear correlations as entirely responsible for quenching, again within uncertainties.

The inequality $S_- = 3(N - Z) + S_+ \geq 3(N - Z)$ is often used to establish the discrepancy between the measured S_- and the theoretical bound, and it is also argued that S_+ is likely to be small in nuclei of large neutron excess, when it originates in correlation terms (i.e. r-components outside the model space). In such cases S_+ may be difficult to measure rather than small and the same could be said of the term $(S_- - S_+)_r$ in eq.(4), always entirely due to correlations. Therefore we propose a statement that is both consistent with the sum rule and with observations

$$(S_- - S_+)_m = (0.7) \cdot 3(N - Z)$$

where m stands conveniently for *model* and *measured*.

It is seen that the quenching and missing strength factors are identical if we assume that all strength due to correlations is missing and all strength coming from the model space is measured. As we shall see now, experiments probably miss most of the former and substantial amounts of the latter.

III. To decide which is the observation threshold for $^{48}\text{Ca}(p, n)^{48}\text{Sc}$ we note that in the data of Anderson *et al* [10] the strong isolated peak at 2.52 MeV (2.3 in figs. 1,2) collects a strength of 6.8 against 38.7 for all the states in the interval 4.5-14.5 MeV. The ratio of the two numbers is 5.7 while we would find 8.4 from fig. 2. In table 1 we show the amount of surviving strength as a function of the threshold. Selecting a cutoff of 0.75% of the total S_-

we obtain fig. 6 where the ratio is now 5.7. It is interesting to compare with fig. 1, which is the one we would have normally kept, and whose relatively modest lowest peak becomes now the largest, in line with what is seen experimentally [10], [25]. The reader is invited to check (or to believe) that the two smallest isolated, observed bumps correspond exactly with the peaks at 3.5 and 5.5 MeV that have (barely) survived the cutoff. This is a very direct indication that the threshold chosen is indeed realistic. In addition to these bumps, the data show three gross structures centered at 7.5, 10 and 12 MeV that correspond closely to what is seen in fig. 6 (ref [25] contains a good plot of the original data [10]).

From the arguments we have presented, it follows that in the $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ experiment, some 25% of the model strength goes unobserved. It is quite plausible to assume that the strength associated to correlations will be spread among smallish peaks and that few of them will survive the cutoff. A $^{48}\text{Ca}(n,p)^{48}\text{K}$ experiment will be very welcome even if very little is seen, to confirm the $S_- - S_+$ loss factor $(0.6-0.7)(0.75)$ i. e. some 50%. It is important to note that this value is not expected to be typical. In fig. 5 we find that only 19% of the strength is located in peaks that survive a 1% cutoff and even the more generous 0.5% will only spare 35% of the strength. Such situations will arise whenever the resonance moves to regions of high level density where dilution is severe. Conversely, for low level densities we may recover the standard factor. It is also of interest to stress that there is no reason to expect that a pile up of small peaks below threshold in a narrow bin of energy could produce a legitimate signal. In this sense, tails and background should be excluded from the collection of strength unless good reasons could be given that they contain large enough peaks. A remark worth doing in this context is that the smallest isolated peak detected in any measure we have consulted contains approximately 1% of the total strength [26].

To conclude. The quenching problem can be solved by invoking deep correlations. The result was either known, or suspected or believed [27] but we think the simple proof presented here may be new and has the advantage of relating the standard factor $\sqrt{0.6}$ to occupancies at the Fermi level. (Again, this was probably guessed by somebody). Missing strength is

another matter. It is certainly there, mostly in the region of the resonance (for model states at least) but its identification will demand an extra effort. Since experimentalist have done already quite extraordinary things it may be unfair -but not hopeless- to demand more.

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TABLES

cutoff(%)	I=50	I=300	I=700
0.05	99.99	99.14	97.85
0.10	99.94	98.57	96.43
0.20	99.45	97.10	93.15
0.50	97.92	89.76	83.55
0.75	97.35	86.98	73.37
1.00	96.36	81.76	67.95

TABLE I. $^{48}\text{Ca}(p,n)^{48}\text{Sc}$: percentage of strength located in peaks whose share of the total strength is larger than the cutoff, as a function of the number of Lanczos iterations.

FIGURES

FIG. 1. $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ Gamow Teller strength function: 50 iterations Lanczos. B(GT) in % of the total strength.

FIG. 2. $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ Gamow Teller strength function: 700 iterations Lanczos. B(GT) in % of the total strength.

FIG. 3. $^{48}\text{Mn} \rightarrow ^{48}\text{Cr}$ Gamow Teller strength function: 50 iterations Lanczos. B(GT) in % of the total strength.

FIG. 4. $^{48}\text{Mn} \rightarrow ^{48}\text{Cr}$ Gamow Teller strength function: 300 iterations Lanczos. B(GT) in % of the total strength.

FIG. 5. $^{48}\text{Mn} \rightarrow ^{48}\text{Cr}$ Gamow Teller strength function: 300 iterations Lanczos. B(GT) in % of the total strength. Only states carrying less than 1 % of the total strength are plotted.

FIG. 6. $^{48}\text{Ca}(p,n)^{48}\text{Sc}$ Gamow Teller strength function: 700 iterations Lanczos. B(GT) in % of the total strength. Only states carrying more than 0.75 % of the total strength are plotted.